Marwari college Darbhanga

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Topic--- Transport of momentum ; Lacture series---13

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Transport of Momentum

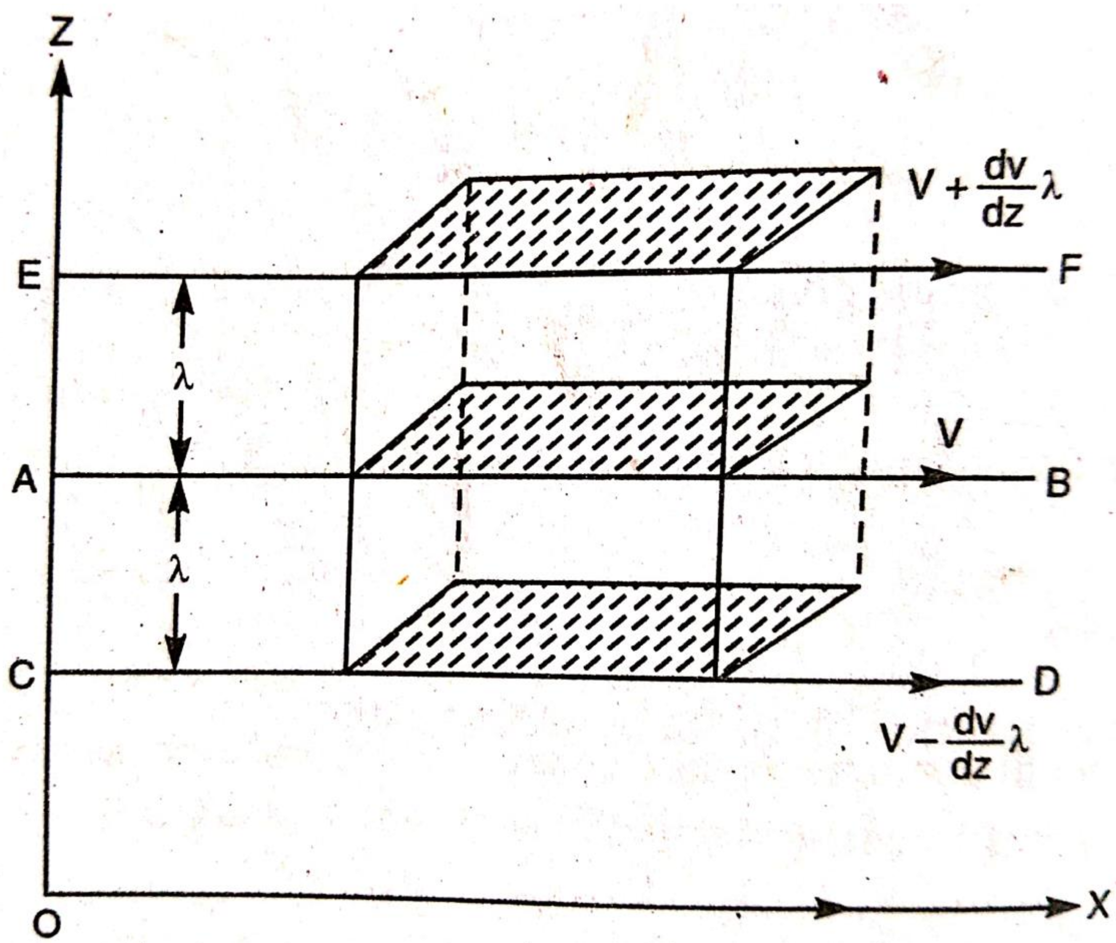

Fig. 01

Consider a gas flowing over a horizontal Surface 'OX' from left to right. The velocity of the layer in contact with the Surface 'OX' is zero and goes on increasing along 'OZ' at a uniform rate dv/dz, known as velocity gradient (from fig.).

Let us consider a layer AB moving with a drift velocity V, situated at a distance Z from O. All the molecules in this layer have the same drift velocity.
Consider layers EF and CD just above and below AB respectively at a distance 'λ' equal to the mean free path of the molecules, so that the molecules moving vertically up or down do not suffer any collision while moving between the two layers.

The velocity of the gas in the layer EF

$$= V + \frac{dv}{dz}\lambda$$

and the velocity of the gas in the layer

$$CD = V - \frac{dv}{dz}\lambda$$

Due to thermal agitation, the gas molecules are moving in all directions, thus, the average number of molecules along any one (+ve or −ve) axis will be $1/6^{th}$ of the total number of molecules in the gas.

Let n = no. of molecules per unit vol.
m = The mass of gas molecule.

C = The average velocity of a molecule at a given temperature of the gas.

Then number of molecules passing downwards from EF to CD per unit area of the AB in one second $= \frac{nc}{6}$

$\therefore$ Forward momentum lost per unit area per second by the layer EF.

$$= m \times \frac{nc}{6} \times \left(v + \frac{dv}{dz}\lambda\right)$$

Similarly, the numbers of molecules passing upwards from CD to EF per unit area of the layer AB in one second is also

$$= \frac{nc}{6}$$

$\therefore$ Forward momentum gained per unit area per second by the layer EF.

$$= m \times \frac{nc}{6} \times \left(v - \frac{dv}{dz}\lambda\right)$$

$\therefore$ Net momentum lost by the layer EF per unit area per second.

$$= \frac{mnc}{6}\left\{\left(v + \frac{dv}{dz}\lambda\right) - \left(v - \frac{dv}{dz}\lambda\right)\right\}$$

$$= \frac{1}{3} mnc\lambda \frac{dv}{dz}$$

The layer CD below AB gains the same amount of momentum. Hence, the layer EF above AB tends to accelerate its motion and the layer CD below AB tends to retard its motion.

The backward dragging force per unit area = gain or loss of momentum per unit area per sec.

$$F = \frac{1}{3} mn\acute{c}\lambda \frac{dv}{dz} \quad \text{——— (1)}$$

This must be equal to the viscous or tangential force $\left(\eta \frac{dv}{dz}\right)$ acting per unit area of the layer AB,

η being the coefficient of viscosity of the gas.

$$\eta \frac{dv}{dz} = \frac{1}{3} mnc\lambda \frac{dv}{dz}$$

$$\Rightarrow \eta = \frac{1}{3} mnc\lambda \quad \text{——— (2)}$$

$$\Rightarrow \boxed{\eta = \frac{1}{3} \rho c\lambda}$$

where $\rho = mn$ = density of ga[s]

* Effect of temperature on 'η'

The density of the gas decreases with increas[e] in temperature but λ, the mean free path increases in the same proportion so that $\rho\lambda$ remains constant.

Since the average molecular speed 'c' is directly proportional to the square root of its absolute temp. $(c \propto \sqrt{T})$, the coefficient of viscosity (η) will also be proportional to $\sqrt{T}$.

$$\therefore \boxed{\eta \propto \sqrt{T}} \quad \text{——— (3)}$$

* Effect of pressure on 'η'

The density of a gas incrases with increase in pressure but λ, the mean free path decreases in the same proportion; so that $\rho\lambda$ remains constant. Moreover, the average moleculas speed 'C' is independent of pressure.

Thus, using relation

$$\eta = \frac{1}{3} \rho C \lambda$$

η, the coefficient of viscosity is independent of pressure.